

# Bias in Reduced-Form Estimates of Pass-through

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## Abstract

We show that, in general, consistent estimates of cost pass-through are not obtained from reduced-form regressions of price on cost. We derive a formal approximation for the bias that arises under standard orthogonality conditions. We provide guidance on the conditions under which bias may frustrate inference.

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# 1 Introduction

This paper addresses the conditions under which cost pass-through can be estimated accurately with reduced-form regressions of price on costs. Our interest in this subject follows recent articles that develop the theoretical properties of pass-through (e.g., Weyl and Fabinger (2013); Fabinger and Weyl (2012)). It is now understood that pass-through can be used to infer structural demand parameters when demand estimation is infeasible (Miller, Remer, and Sheu (2013)), to make counterfactual predictions without functional form restrictions (Jaffe and Weyl (2013); Miller, Remer, Ryan, and Sheu (2013)), and to evaluate the magnitude of market power (Scharfstein and Sunderam (2013)). The empirical literature on pass-through, on which our results have direct bearing, is substantial and spans the fields of industrial organization (e.g., Borenstein, Cameron, and Gilbert (1997); Fabra and Reguant (2013)) and international trade (e.g., Atkeson and Burstein (2008); Gopinath, Gourinchas, Hsieh, and Li (2011)).

Our main result is that a reduced-form regression of price on costs – the standard methodology for pass-through estimation – only yields a consistent estimate if the underlying economic model has specific properties. The usual assumption that observed cost measures are uncorrelated to other determinants of cost does not guarantee consistency.

We derive a second-order approximation for the bias under the assumption that the regressor is uncorrelated with unobserved costs. For all twice differentiable pricing functions, including those that arise from Bertrand profit-maximizing behavior, bias can be decomposed into two components. The first component is due to regression misspecification and occurs if the distribution of costs is skewed and pass-through varies with costs. Misspecification bias is present even if all variables are observed perfectly and, in isolation, can be accounted for via standard techniques (such as a polynomial regression or splines). The second component, which we call “partial information bias,” arises if the marginal costs of a firm or its competitors are partially observed, pass-through varies with costs, and the observed and unobserved costs are not independent. Because independence can be a strong assumption, we also provide bounds for partial information bias that can be calculated given information on the underlying demand system and plausible assumptions on the distribution of costs.

Neither misspecification bias and nor partial information bias arise if the underlying economic model is characterized by constant pass-through. Thus, the standard methodology for pass-through estimation can be motivated by invoking constant pass-through models, such as those developed in Bulow and Pfleiderer (1983).<sup>1</sup> Alternatively, because pass-through

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<sup>1</sup>Constant pass-through occurs in a class of demand systems that includes the linear, log-linear, and

is not constant generally, the standard methodology can be motivated by invoking symmetric cost distributions and independence between observed and unobserved costs. To our knowledge, prior research has not fully recognized the conditions under which misspecification bias and partial information bias arise. Thus, our result will be useful to researchers developing new empirical estimates of pass-through or seeking to interpret the existing literature.

We introduce the two sources of bias by way of numerical example in Section 2. We then develop the main theoretical result in Section 3 and provide discussion. Bounds to partial information bias are developed in Section 4.

## 2 Numerical Examples

Consider first the case of a monopolist facing a logit demand schedule. Let the mean consumer valuation of the monopolist's good equal two minus the monopolist's price, and let the mean consumer valuation of the outside option be zero. This gives rise to the demand schedule

$$s = \frac{\exp(2 - p)}{1 + \exp(2 - p)},$$

where  $s$  is the monopolist's market share and  $p$  is its price. Theoretical pass-through is not constant with logit demand and, in the case of monopoly, equals  $1 - s$ . Let the monopolist's marginal costs be drawn from a distribution with expected value 0.5 and variance 0.083. At the expected marginal cost, the profit maximizing price yields a market share for the monopolist of 0.434 and a pass-through of 0.566.

Suppose that an econometrician has 100,000 observations of costs and the associated profit maximizing prices. Will a reduced-form estimate obtain a meaningful parameter? We simulate results drawing costs from a uniform distribution and a lognormal distribution.<sup>2</sup> The reduced-form regression yields a nearly precise estimate of theoretical pass-through in the case of uniformly distributed costs. The lognormal distribution, by contrast, results in a pass-through estimate of 0.604, roughly 6.5% higher than the actual pass-through rate at the expected cost. The source of this bias, as we detail below, is misspecification of the reduced-form regression equation combined with an asymmetric cost distribution and pass-through that is not constant.

Now suppose instead that the econometrician observes only a fraction of the monop-

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constant-markup demand systems.

<sup>2</sup>Specifically we use a uniform distribution on  $[0, 1]$  and a log-normal distribution with parameters  $\sigma = \sqrt{\ln(4/3)}$  and  $\mu = \ln(1/2) - (1/2)\sigma^2$ .

olist's marginal cost. The observed component, which we denote  $c_1$ , has a uniform distribution with expected value 0.5 and variance 0.083. The unobserved component takes the form  $c_2 = Rc_1$  where  $R$  is a Rademacher weight that takes the values of one and negative one with equal probability. Thus the observed and unobserved components of cost, while uncorrelated, are not independent because  $c_1$  is correlated with the conditional variance of  $c_2$ . Again the econometrician obtains 100,000 observations of profit-maximizing prices and observed costs, and uses a univariate reduced-form regression to estimate pass-through. Our simulations indicate a point estimate of 0.642, which is 12.4% higher than the theoretical pass-through at the expected cost.<sup>3</sup> We refer to this bias as partial information bias because the source is the the unobserved cost component, which is missing from the regression equation. We view this as distinct from omitted variable bias, which typically is derived based on correlation between a regressor and the error term in linear settings.

### 3 Theoretical Result

We provide a theoretical result that explains the numerical results developed above, in the form of the following proposition:

**Proposition 1:** *Let the equilibrium price of a firm take the form  $p = f(c_1, c_2)$  where  $f$  is a twice differentiable pricing function and  $c_1$  and  $c_2$  are stochastic cost terms with expected values  $a$  and  $b$ , respectively. Further let  $c_1$  and  $c_2$  be mean independent so that  $E[c_2|c_1] = b$ . Then, to a second-order approximation and for any data generating process, the probability limit of the coefficient obtained from a univariate regression of  $p$  on  $c_1$  equals*

$$\text{plim}\hat{\rho} = f_1(a, b) + \frac{1}{2}f_{11}(a, b)\frac{E[(c_1 - a)^3]}{\text{Var}(c_1)} + \left( f_{12}(a, b)\frac{\text{Cov}((c_1)^2, c_2)}{\text{Var}(c_1)} + \frac{1}{2}f_{22}(a, b)\frac{\text{Cov}(c_1, (c_2)^2)}{\text{Var}(c_1)} \right),$$

where  $f_i$  is the partial derivative of  $f$  with respect to  $c_i$  and  $f_{ij}(a, b)$  is the second derivative of  $f$  with respect to  $c_i$  and  $c_j$ .

**Proof:** See the Appendix.

The pricing function  $f$  can be conceptualized as the equilibrium strategy for a consumer demand schedule and a competitive game. Thus, it is fully consistent with oligopoly price theory. The first term on the right hand side of the main equation,  $f_1(a, b)$ , is the theoretical pass-through that arises at the expected value of the cost distribution. We take as given

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<sup>3</sup>The standard errors with 100,000 observations are very small.

that this is the object of interest, although in some settings the researcher may be interested in other notions of pass-through.

The second term is the misspecification bias that arises due to the curvature of the pricing function. There is no misspecification bias if either (i) pass-through is constant, in which case  $f_{11}(a, b) = 0$ , or (ii) the distribution of  $c_1$  is symmetric such that the third central moment  $E[(c_1 - a)^3]$  is zero. Indeed, misspecification bias explains the result of the lognormal example in Section 2. Given the parameterization employed, it is possible to calculate that with log-normally distributed costs  $\frac{1}{2}E[(c_1 - a)^3]/Var(c_1) = 0.266$ . This component times the second derivative of the pricing function gives a value of 0.037, which is close to the empirical bias of 0.038 that arises with the reduced-form regression.

If misspecification is the only source of bias then the regression coefficient represents a weighted average of the theoretical pass-through that arises at the realized cost draws. While this object can be useful to empirical researchers in some settings, its use also can hinder inference in studies that treat pass-through as an outcome of interest. As one example, consider an econometrician that seeks to compare pass-through across markets or over time as an indicator for changing market power (e.g., as in Scharfstein and Sunderam (2013)). If the true economic model features non-constant pass-through and an asymmetric cost distribution, and if the realized cost draws vary across markets, then reduced-form regression results can be unreliable. Misspecification bias in isolation can be accounted for via standard techniques (e.g., splines or polynomials), but if partial information bias is also present then these adjustments can confound rather than improve inference.

The final term, in parentheses, is the partial information bias that arises from the exclusion of  $c_2$  from the reduced-form regression. There is no partial information bias if pass-through is constant. If pass-through is not constant then the standard assumption that  $c_1$  and  $c_2$  are uncorrelated is insufficient to eliminate partial information bias.<sup>4</sup> For example, pass-through estimates are biased upward if  $c_1$  is positively correlated with the conditional variance of  $c_2$  and the pricing function is convex, as is the case with the example in Section 2. Partial information can arise in a variety of settings because  $c_2$  can be thought of either as a marginal cost component or as an unobserved cost of a competitor. If  $c_2$  is an unobserved cost term, then it affects pass-through via total marginal costs. If instead  $c_2$  is a competitor's costs, then it influences the competitor's price and affects pass-through indirectly.

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<sup>4</sup>Intuitively, it may seem that if the variance of  $c_1$  is small, the approximation is local, and therefore the impact of higher-order moments is limited. However, this is not the case, as both  $Cov((c_1)^2, c_2)/Var(c_1)$  and  $Cov(c_1, (c_2)^2)/Var(c_1)$  may be large as  $Var(c_1)$  approaches zero.

## 4 Bounds on Partial Information Bias

The condition that  $c_1$  and  $c_2$  are independent is sufficient to eliminate partial information bias. When independence does not hold it is possible to bound the magnitude of bias, given an assumption on the pricing function (which implies a demand system and competitive game) and restrictions on the distribution of unobserved costs. The pricing function allows the terms  $f_{12}(a, b)$  and  $f_{22}(a, b)$  to be obtained. Then, recognizing that  $Cov(c_1, (c_2)^2)/Var(c_1)$  is the slope coefficient from a regression of  $(c_2)^2$  on  $c_1$ , we have that

$$\frac{Cov(c_1, (c_2)^2)}{Var(c_1)} < \frac{\max_{c_1} E[(c_2)^2|c_1] - \min_{c_1} E[(c_2)^2|c_1]}{c_1^{max} - c_1^{min}}.$$

If we further assume that  $E[c_1|c_2] = E[c_1]$ , then this becomes

$$\frac{Cov(c_1, (c_2)^2)}{Var(c_1)} < \frac{\max_{c_1} Var(c_2|c_1) - \min_{c_1} Var(c_2|c_1)}{c_1^{max} - c_1^{min}}.$$

Reasonable guesses for the range of the conditional variance in the unobserved component of cost will generate bounds for the bias. The other component of bias can be bounded similarly, by recognizing that

$$\frac{Cov((c_1)^2, c_2)}{Var(c_1)} = \frac{Cov((c_1)^2, c_2)}{Var(c_2)} \frac{Var(c_2)}{Var(c_1)} < \frac{\max_{c_2} Var(c_1|c_2) - \min_{c_2} Var(c_1|c_2)}{c_2^{max} - c_2^{min}} \frac{Var(c_2)}{Var(c_1)}.$$

Thus, researchers sometimes may be able to assess the reliability of reduced-form regression even in economic environments with non-constant pass-through and an unobserved marginal cost term that is not independent from the observed cost component.

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# Appendix

## A Proof of Proposition 1

The proof is by construction. Consider the second-order Taylor expansion of the equilibrium price around the expected costs  $(a, b)$ :

$$\begin{aligned} p &\approx f(a, b) + f_1(a, b)(c_1 - a) + f_2(a, b)(c_2 - b) \\ &\quad + \frac{1}{2} [f_{11}(a, b)(c_1 - a)^2 + 2f_{12}(a, b)(c_1 - a)(c_2 - b) + f_{22}(a, b)(c_2 - b)^2] \\ &= [\text{constant}] + f_1(a, b)c_1 + \frac{1}{2}f_{11}(a, b)((c_1)^2 - 2ac_1) + f_{12}(a, b)(c_1c_2 - ac_2 - bc_1) \\ &\quad + f_2(a, b)c_2 + \frac{1}{2}f_{22}(a, b)((c_2)^2 - 2bc_2) \end{aligned}$$

The plim of  $\hat{p}$  from a regression of  $p$  on  $c_1$  is equal to  $\frac{Cov(p, c_1)}{Var(c_1)}$ , where

$$\begin{aligned} Cov(p, c_1) &\approx f_1(a, b)Var(c_1) - af_{11}(a, b)Var(c_1) - bf_{12}(a, b)Var(c_1) \\ &\quad + \frac{1}{2}f_{11}(a, b)Cov(c_1, (c_1)^2) + f_{12}(a, b)Cov(c_1, c_1c_2) + \frac{1}{2}f_{22}(a, b)Cov(c_1, (c_2)^2). \end{aligned}$$

Therefore it follows that

$$\begin{aligned} \text{plim}\hat{p} &\approx f_1(a, b) + f_{11}(a, b) \left[ \frac{1}{2} \frac{Cov(c_1, (c_1)^2)}{Var(c_1)} - a \right] \\ &\quad + \left( f_{12}(a, b) \left[ \frac{Cov(c_1, c_1c_2)}{Var(c_1)} - b \right] + \frac{1}{2}f_{22}(a, b) \frac{Cov(c_1, (c_2)^2)}{Var(c_1)} \right), \end{aligned}$$

and this yields Proposition 1 after minor algebraic manipulations.