Using Cost Pass-Through to Calibrate Demand*

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November 27, 2012

Abstract

We demonstrate that cost pass-through can be used to inform demand calibration, potentially eliminating the need for data on margins, diversion, or both. We derive the relationship between cost pass-through and consumer demand using a general oligopoly model of Nash-Bertrand competition and develop specific results for four demand systems: linear demand, logit demand, log-linear demand and the Almost Ideal Demand System (AIDS). The methods we propose may be useful to researchers and antitrust authorities when reliable measures of margins or diversion are unavailable. We also develop that cost pass-through can help illuminate the suitability of some demand systems to specific economic applications.

Keywords: cost pass-through; demand calibration; merger simulation
JEL classification: K21; L13; L41

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*The views expressed herein are entirely those of the authors and should not be purported to reflect those of the U.S. Department of Justice.

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1 Introduction

Researchers in industrial economics frequently conduct counter-factual experiments based on parameterized systems of consumer demand. The functional form of demand is assumed and the structural parameters are either estimated from data or calibrated. Our focus in this paper is on demand calibration. Heretofore, calibration for differentiated product industries has been thought to require information on price-cost margins and consumer diversion, which together are sufficient to recover the structural parameters of many demand systems.[1]

We develop that cost pass-through can be used to inform demand calibration, potentially obviating the need for margins, diversion, or both. As a motivating example, suppose an economist seeks to calibrate a linear demand system to facilitate merger simulation. Price-cost margins are available so the own-price elasticities of demand are obtainable through first order conditions. Unfortunately, the available data are insufficient to estimate diversion and the documentary evidence is unhelpful. We demonstrate that cross-price elasticities nonetheless can be selected to rationalize cost pass-through, perhaps obtained from documents or estimated with reduced-form regressions of prices on cost shifters. In this example, cost pass-through replaces information on diversion in the calibration process.

The connection between cost pass-through and the properties of demand has been emphasized in the recent theoretical literature. Jaffe and Weyl (2012) propose using cost pass-through to inform the second order properties of demand (i.e., demand curvature) given knowledge of the first order properties (i.e., demand elasticities).[2] Our findings flip the intuition. Cost pass-through can inform the first order properties of demand provided the economist is willing to assume the functional form of demand and thereby fix the second order properties. In the motivating example of linear demand, cost pass-through is informative because the cross-elasticities of demand for any two products relate to the degree to which the products' prices are strategic compliments, in the sense of Bulow, Geanakoplos, and Klemperer (1985), which in turn relates to cost pass-through.

Our findings parallel the contemporaneous work of Atkin and Donaldson (2012), which uses an assumption on the second order properties of demand to estimate pass through and then back out certain terms entering in the firms' first order conditions (namely, marginal

[1]Consumer diversion from one product to another can be defined as the proportion of consumers leaving the first product, in response to a small price increase, that switch to the second product. Knowledge of quantities and prices is also necessary for calibration. Relative to demand estimation, calibration is more common among antitrust practitioners because it can utilize confidential information that becomes available to the U.S. Department of Justice and the Federal Trade Commission under the Hart-Scott-Rodino Act.

[2]See also the discussion in Miller, Remer, Ryan, and Sheu (2012).

costs). Here we leverage the same relationship between pass-through and the first order conditions to identify demand parameters in other settings.

The paper proceeds in three parts. We first derive the relationship between cost pass-through and the properties of demand in a general oligopoly model of Nash-Bertrand competition, following Jaffe and Weyl (2012). We then develop how cost pass-through can inform the calibration of four specific demand systems: linear demand, logit demand, log-linear (or isoelastic) demand, and the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980). These systems are commonly employed in antitrust analysis of mergers involving differentiated products (e.g., Werden, Froeb, and Scheffman (2004); Werden and Froeb (2008)). The results provide methods of calibration that are useful when reliable measures of margins or diversion are unavailable, but when cost pass-through can be estimated from data or discerned from other sources. Furthermore, we demonstrate the extent to which observed cost pass-through rates can provide information on the true underlying demand system, which is valuable because merger simulation can be sensitive to demand specifications (Crooke, Froeb, Tschantz, and Werden (1999); Miller, Remer, Ryan, and Sheu (2012)). Finally, we provide a numerical example based on the linear demand system.

2 General Model of Cost Pass-Through

Consider a model of Nash-Bertrand competition in which firms face well-behaved and twice-differentiable demand functions. Each firm i sells a single product and sets prices to maximize short-run profits, taking as given the prices of its competitors.[3] The first order conditions that characterize firm i's profit-maximizing prices can be expressed

$$f_i(P) \equiv - \left[\frac{\partial Q_i(P)}{\partial P_i}^T \right]^{-1} Q_i(P) - (P_i - MC_i) = 0, \tag{1}$$

where Q_i is a vector of firm i's sales, P_i is a vector of firm i's prices, P is a vector of all prices, and MC_i is a vector of firm i's marginal cost.

Now suppose that a per-unit tax is levied on each product in the model – the tax perturbs marginal costs and allows for the derivation of cost pass-through. The post-tax first order conditions are

$$f(P) + t = 0,$$

[3]We restrict attention to single-product firms to simplify exposition; our results extend to the multi-product case.

where t is the vector of taxes and $f(P) = [f_1(P)'\ f_2(P)'\ \ldots]'$. Differentiating with respect to t obtains

$$\frac{\partial P}{\partial t}\frac{\partial f(P)}{\partial P} + I = 0,$$

and algebraic manipulations then yield

$$\frac{\partial f(P)}{\partial P} = -\left(\frac{\partial P}{\partial t}\right)^{-1}. \tag{2}$$

Thus, the Jacobian of $f(P)$ equals the opposite inverse of the cost pass-through matrix. This Jacobian depends on both the first and second derivatives of demand, as can be ascertained from equation 1, and it follows that cost pass-through similarly relates to both the first order and second-order properties of demand.[4]

3 Cost Pass-Through with Specific Demand Systems

The premise of this paper is that, provided one is willing to specify the functional form of demand, expression (2) can be used to calibrate the structural demand parameters, i.e., to select demand parameters that rationalize observed cost pass-through.[5] At most there are J^2 demand elasticities to be identified, and expression (2) provides J^2 equations with which to work. We examine four specific demand systems: linear demand, logit demand, log-linear demand and the AIDS. We show how the linear, logit and log-linear demand systems can be calibrated with cost pass-through and either margins or diversion – the demand parameters of these systems are under-identified with cost pass-through alone. By contrast, AIDS can be calibrated solely with cost pass-through in many cases. We also develop that cost pass-through can inform the suitability of some demand systems in research or policy applications, as each of the aforementioned demand systems cannot be rationalized by some subset of cost pass-through matrices. Thus, measures of cost pass-through can help inform whether these demand systems are consistent with consumer behavior in the industry at question.

[4] See Miller, Remer, Ryan, and Sheu (2012) for an explicit derivation of $\partial f(P)/\partial P$.

[5] Precise calibration requires knowledge of all cost pass-through rates because each element of $\partial f(P)/\partial P$ depends on all the elements of $\partial P/\partial t$.

3.1 Linear Demand

The linear demand system has the form

$$Q_i = \alpha_i + \sum_j \beta_{ij} P_j. \tag{3}$$

The parameters to be calibrated include J product-specific intercepts and J^2 price coefficients. The elements of the Jacobian of $f(P)$ are

$$\frac{\partial f_i(P)}{\partial P_j} = \begin{cases} -2 & \text{if } i = j \\ -\beta_{ij}/\beta_{ii} & \text{otherwise.} \end{cases} \tag{4}$$

Cost pass-through identifies at most $J \times (J-1)$ price coefficients in this case because the Jacobian of $f(P)$ has constants along the diagonal. To calibrate the demand system, the own-price coefficients can be inferred from margins and the firms' first order conditions.[6] The cross-price coefficients then can be identified using cost pass-through and equation (2), and the intercepts can be recovered from price and quantity data. The linear demand system can be calibrated with cost pass-through and margins. Cost pass-through thereby relieves the need for diversion. Equation (4) also yields the insight that a cost pass-through matrix can be rationalized by some parameterization of linear demand if and only if the diagonal elements of its opposite inverse equal negative two.

3.2 Logit Demand

The logit demand system has the form

$$Q_i = \frac{e^{(\eta_i - P_i)/\tau}}{\sum_k e^{(\eta_k - P_k)/\tau}} M, \tag{5}$$

where M is the size of the market and the parameters include J product-specific terms (η_i for $i = 1 \ldots, J$) and a single scaling/price coefficient (τ). It is standard to normalize the market size to one, so that quantities can be interpreted as market shares. The elements of the Jacobian of $f(P)$ are

$$\frac{\partial f_i(P)}{\partial P_j} = \begin{cases} -\frac{M}{M-Q_i} & \text{if } i = j \\ \frac{Q_i Q_j}{(M-Q_i)^2} & \text{otherwise} \end{cases} \tag{6}$$

[6] The first order conditions provide that $\beta_{ii} = -\frac{Q_i}{P_i}\frac{1}{m_i}$ where m_i is the margin.

Logit demand typically is calibrated with the margin of a single product and shares; the former obtains the price coefficient and the latter obtains the product-specific terms, as diversion is proportional to share in the logit framework. Equation (6) shows that cost pass-through can substitute for shares in this procedure. Thus, the logit demand system can be calibrated with cost pass-through and a single margin. Equation (4) also demonstrates that a cost pass-through matrix can be rationalized by some parameterization of logit demand if and only if (i) its opposite inverse has diagonal elements that are negative and exceed one in absolute value and (ii) has off-diagonal elements that are consistent with the diagonal.[7]

3.3 Log-linear Demand

The log-linear demand system takes the form

$$\ln(Q_i) = \gamma_i + \sum_j \epsilon_{ij} \ln P_j. \tag{7}$$

The parameters to be calibrated include J product-specific intercepts and J^2 price coefficients; the price coefficients are the own-price and cross-price elasticities of demand. It can be derived that the elements of the Jacobian of $f(P)$ are

$$\frac{\partial f_i(P)}{\partial P_j} = \begin{cases} -\frac{1+\epsilon_{ii}}{\epsilon_{ii}} & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Cost pass-through identifies the own-price elasticities in this case. The cross-price elasticities can be inferred from diversion, and the intercepts then can be recovered from price and quantity data. Thus, cost pass-through relieves the need for margins. Equation (4) also makes clear that a cost pass-through matrix can be rationalized by some parameterization of log-linear demand if and only if (i) the diagonal elements of its opposite inverse are negative and less than one in absolute value,[8] and (ii) its off-diagonal elements and those of its opposite inverse equal zero. This latter condition reflects the unusual property of log-linear demand that prices are neither strategic substitutes nor strategic complements.

[7]The J product-specific terms are over-identified by the J^2 elements of the cost pass-through matrix.

[8]In Nash-Bertrand competition and log-linear demand, elasticities are always less than negative one when firms are profit-maximizing. Also, note that this identifying restriction is equivalent to own cost pass-through being positive and greater than one. With log-linear demand, the implied own-cost pass-through rate of firm i is given by $\epsilon_{ii}/(1+\epsilon_{ii})$, which is greater than one for any elasticity consistent with profit maximization. This can be obtained by rearranging the first order conditions as $P_i = \left(\frac{\epsilon_{ii}}{1+\epsilon_{ii}}\right) MC_i$, and noting that demand elasticities are constant with log-linear demand.

3.4 AIDS Demand

The AIDS, derived in Deaton and Muellbauer (1980), takes the form

$$W_i = \psi_i + \sum_j \phi_{ij} \log P_j + \beta_i \log(x/P^*), \tag{9}$$

where W_i is an expenditure share (i.e., $W_i = P_i Q_i / \sum_k P_k Q_k$), x is the total expenditure and P^* is a price index given by

$$\log(P^*) = \psi_0 + \sum_k \psi_k \log(P_k) + \frac{1}{2} \sum_k \sum_l \phi_{kl} \log(P_k) \log(P_l).$$

We focus on the special case of $\beta_i = 0$, consistent with common practice in antitrust applications (e.g, Epstein and Rubinfeld (1999)). The restriction is equivalent to imposing an income elasticity of one. The elements of the Jacobian of $f(P)$ are

$$\frac{\partial f_i(P)}{\partial P_j} = \begin{cases} \frac{\phi_{ii}(W_i - 2\phi_{ii})}{(\phi_{ii} - W_i)^2} & \text{if } i = j \\ -\frac{P_i \phi_{ii} \phi_{ij}}{P_j(\phi_{ii} - W_i)^2} & \text{otherwise.} \end{cases} \tag{10}$$

Cost pass-through can be sufficient to identify all J^2 price coefficients. This occurs when the own-price coefficients are obtainable from the diagonal elements of $\partial f(P)/\partial P$. The cross-price coefficients then can be inferred from the off-diagonal elements and the demand intercepts can be inferred from equation (9).[9] Obtaining the own-price coefficients, however, requires solving a quadratic equation that could have multiple solutions or no solution. If multiple solutions exist then additional information, such as margins, can be used to obtain the own-price coefficients. If no solution exists then then the observed cost pass-through rates cannot be rationalized by any parameterization of the AIDS.

[9] This result requires that total expenditure be constant with respect to price changes, an assumption that could be reasonable for applications dealing with small price movements. An alternative approach would be to add an elasticity of total expenditure parameter to the model as an extra unknown to calibrate. This elasticity would appear in both the own-price and cross-price terms in equation (10). Then cost pass-through would identify the price coefficients as a function of the added elasticity. Information on one firm's margin could then be used to recover the added elasticity, making use of the firm's first order condition. This derivation is available on request.

4 An Example

Suppose there are three single-product firms and the following cost pass-through matrix is estimated from data:

$$\frac{\partial P}{\partial t} = \begin{bmatrix} .58 & .15 & .17 \\ .23 & .61 & .20 \\ .21 & .25 & .61 \end{bmatrix}.$$

This cost pass-through matrix can be rationalized by linear demand because the opposite inverse of the matrix has diagonal elements that equal minus two. Further suppose that the unit sales of the three firms are 200, 175, and 150, respectively, that prices are \$10, \$9, and \$8, respectively, and that each firm has a 50% margin. Assuming that demand is in actuality linear, the first order conditions imply that the firms' own-price coefficients are -40, -39, and -38. Placing these own-price coefficients into the Jacobian of $f(P)$, cross-price terms can be selected to equate the Jacobian with the opposite inverse of the pass-through rate matrix. This produces the following price coefficient matrix:

$$\beta = \begin{bmatrix} -40 & 12 & 18 \\ 24 & -39 & 19 \\ 17 & 27 & -38 \end{bmatrix}.$$

Combining this matrix with prices and unit sales, following equation (3), yields demand intercepts of 342, 137, and 45.

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