Modeling the Effects of Mergers in Procurement*

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Abstract

In procurement settings, mergers among suppliers reduce buyers' choice sets and can harm buyers by eliminating their preferred supplier or reducing their negotiating leverage. I develop a stochastic economic model that predicts the effects of mergers based on information that commonly is available to antitrust authorities. I derive general expressions for the *ex ante* expected changes in price, buyer utility, and supplier profit. Each becomes tractable under certain distributional assumptions. The model predicts that average prices will increase by more than 40% due to the recently litigated acquisition of Power Reviews by Bazaarvoice, in the absence of an effective remedy.

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JEL classification: K21; L13; L41

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1 Introduction

Firms engaged in procurement typically evaluate the prices and other contract terms of prospective suppliers in conjunction with non-price considerations such as reputation and quality. When more than one supplier is available, the buyer can play each off the others to obtain more favorable terms of trade. Mergers among suppliers reduce the buyer's choice set and, unless sufficient efficiencies arise, can harm the buyer by eliminating its preferred offer or relaxing a binding constraint on the terms of trade with its preferred supplier.

This theory of harm was employed by the U.S. Department of Justice (DOJ) in its challenge of the consummated acquisition of Power Reviews, Inc. ("Power Reviews") by Bazaarvoice, Inc. ("Bazaarvoice").[1] The products at issue are ratings and review (PRR) platforms that enable clients to collect and display consumer-generated product ratings and reviews online. According to court documents, Bazaarvoice employs "value based pricing" in which contract terms are negotiated based on clients' perceived willingness to pay, taking into account the competitive alternatives. The DOJ alleged, and the district court agreed, that buyers of PRR platforms benefited from the ability to play Bazaarvoice and Power Reviews off each other to obtain improved terms of trade, and that, absent relief, the acquisition would reduce the negotiating leverage of many buyers and possibly lead to the discontinuation of the Power Reviews product.

In this article, I develop a stochastic economic model of procurement that predicts the likely price effects of mergers involving the combination of suppliers in procurement markets. The model can be calibrated with information that commonly is available to antitrust authorities, such as market shares, average prices, and margins. To demonstrate, I glean estimates of the requisite data from court documents and the 2013 Annual Report of Bazaarvoice, and calibrate the model to match the facts of the PRR market. The model predicts average price increases of \$65 (42%) for Bazaarvoice and \$84 for Power Reviews. If instead the Power Reviews product is discontinued, the model predicts an average utility loss of \$79,000 annually for the clients of Power Reviews.[2]

The model itself is a stochastic version of the canonical scoring auction model in which buyers "score" the offers of prospective suppliers and award a contract to the supplier with

[1]Bazaarvoice purchased Power Reviews on June 12, 2012. The parties were not required to file with the antitrust agencies because the revenues of Power Reviews fell just below the Hart-Scott-Rodino thresholds. Nonetheless, the DOJ opened an investigation two days after the transaction closed and filed a Complaint on January 10, 2013, alleging a violation of Section 7 of the Clayton Act. Arguments started in September 2013. The district court ruled in favor of the DOJ on January 8, 2014.

[2]Computer code is posted on the author's personal website.

the highest score (e.g., Laffont and Tirole 1987; Che 1993). Since scoring incorporates price and non-price information, the model is well suited for the competitive interplay that arises in procurement settings. The stochastic element allows for the possibility that buyers disagree about the relative desirability of the suppliers, conveying realism to the model and allowing the model to be calibrated from market-level data.

Prospective suppliers in the model are endowed with (i) a marginal cost of supply and (ii) a value that can be provided to the buyer. The difference between the value and the cost is the surplus that a supplier can create. I treat surplus as an exogenous stochastic variable that varies over suppliers and auctions. This departs from the theoretical literature on scoring auctions, which typically treats value as a choice variable of suppliers (e.g., Laffont and Tirole 1987; Che 1993; Branco 1997; Ganuza and Pechlivanos 2000; Asker and Cantillon 2008, 2010). The restriction avoids complications due to the potential non-contractablity of quality (Klein and Leffler 1981; Taylor 1993; Che and Gale 2003) and the impact of moral hazard and renegotiation (Bajari and Tadelis 2001; Bajari, McMillan and Tadelis 2009).

In this context, the outcome of the scoring auction and the transaction price that arises is fully determined by the surplus created by the suppliers. I derive general expressions for the *ex ante* expected changes in price, buyer utility, and supplier profit that arise due to a merger between two suppliers. I consider separately the cases in which (i) the merged entity retains both of its products post-merger and (ii) the merged entity discontinues one of its products. In practice, whether mergers result in product discontinuation depends heavily on the fixed costs of maintaining a product line, which are outside the model developed here. I then show how the general expressions can be made tractable by invoking a stochastic process for surplus that falls either within the location-scale or power-related families of distributions. Closed-form expressions are available when surplus arises from the Gumbel distribution.

The prices are set so that buyer utility, in a transaction with the highest-surplus supplier, just exceeds the surplus that could have been created by the second-best supplier. This "second-score" approach is strategically equivalent to a specific form of bargaining in which buyers play suppliers off against each other, up to the point at which the utility offered by the highest-surplus supplier cannot be matched profitably by the next best supplier. At that point, buyers have no more leverage and negotiations end. This conveys tractability to the analysis and is reasonable for many markets with either (i) merging suppliers that are much larger than most buyers, or (ii) buyers that have sizable negotiating costs. If buyers exercise leverage above-and-beyond the competitive alternative, then the fit of the model is less good. Nonetheless, I show that the model generalizes easily to Nash-in-Nash bargaining (e.g., Horn and Wolinsky 1988), which features more extensive bargaining between buyers and the highest-surplus suppliers. I also develop results for first-score auctions, which feature none of the "back-and-forth" between buyers and sellers that characterize procurement markets, and draw analogies to Nash-Bertrand pricing in consumer products markets.

The model nests as special cases a number of previously developed models that examine mergers among undifferentiated suppliers with heterogeneous production costs. For instance, Froeb, Tschantz and Crooke (1999) and Waehrer and Perry (2003) develop results based on power-related distributions and Froeb, Tschantz and Crooke (1998, 2000) develop results based on location-scale distributions. That these results generalize to conditions with differentiated suppliers is anticipated by the theoretical literature on scoring auctions (e.g., Che 1993), on the basis that surplus fully characterizes equilibrium outcomes: it is immaterial whether heterogeneity in surplus arises from cost heterogeneity alone or from heterogeneity in both cost and value. Thus, a primary contribution of this article is that it unifies the applied antitrust literature on auctions, which emphasizes stochastic elements in undifferentiated settings, with the theoretical literature on scoring auctions, which is more general but lacks the stochastic elements that are necessary for calibration.

The model also can be related to a growing empirical literature on auctions and negotiations when supplier quality matters (e.g., Lewis and Bajari 2011; Grennan 2013; Gowrisankaran, Nevo, and Town 2014; Krasnokutskaya, Song, and Tang 2014). These articles use structural techniques, supported by large datasets, to (i) recover the supplier qualities that rationalize the data; and (ii) identify buyer tastes for price and quality.[3] Grennan (2013) is of particular relevance because it considers mergers, albeit on the buyer-side of procurement, in the context of the market for medical devices. The emphasis here is complementary, in the sense that the model facilitates analysis of supply-side mergers with much less data, relying on the scoring rule formulation to create a unidimensional choice problem. That said, I show that it is straight-forward to extend the model to accommodate arbitrary patterns of customer substitution, along the lines of Berry, Levinsohn, and Pakes (1995) and Nevo (2001), which could provide a useful framework for future empirical research.

Important caveats apply. First, the model leverages distributional assumptions to make predictions based on limited data. These assumptions help determine the magnitude of predicted price changes, as they do in consumer products settings (Crooke, Froeb, Tschantz and Werden 1999; Miller, Remer, Ryan and Sheu 2013). I discuss in the text how, with increas-

[3]These articles treat quality as exogenous. The exception is Lewis and Bajari (2011), which examines highway repair contracts in California over 2003-2008. This allows for a richer strategic environment but also places more demands on the variation that must be present in the data.

ing amounts of data, these assumptions can be checked or relaxed. Second, the model treats supplier value as exogenous. This eliminates interesting strategic interactions that otherwise could arise. How predictions would be affected by relaxing this assumption is a matter I leave to future research. Third, I assume throughout that buyers have perfect information about surplus. While this treatment is consistent with the published literature, recent research also examines scoring auctions in which buyers are imperfectly informed (Giebe and Schweinzer 2014). Finally, the application to the Bazaarvoice/Power Reviews merger gleans information on average prices and costs from accounting reports of Bazaarvoice. The reported price effects are subject to the additional caveat that it is difficult to assess whether these inputs are appropriate based only on publicly-available information.

The article proceeds as follows. Section 2 contains a discussion of merger enforcement in procurement auctions. Section 3 develops notation and sketches the model. Section 4 derives general expressions for the effects of mergers on market outcomes. Section 5 shows how employing distributions from location-scale and power-related families can sharpen predictions, and provided closed-form solutions for the case of the Gumbel distribution. Section 6 provides extensions to Nash-in-Nash bargaining and first-score auctions. Section 7 concludes.

2 Merger Enforcement in Procurement Markets

It generally is thought that the overall volume of business-to-business transactions far exceeds that of business-to-consumer transactions (e.g., Sandhusen 2008) because supply-chains typically feature multiple exchanges between businesses, but only a single final sale. Of the three fully litigated merger challenges by the DOJ since 2000, two involved business-to-business procurement markets. The most recent, Bazaarvoice/Power Reviews, is discussed in some detail above. The other, Oracle/Peoplesoft, concerned the sale of high-functionality software for human resources and financial management to large business and government customers.[4] The DOJ challenge of AT&T/T-Mobile similarly involved, in part, sales to large enterprise customers. In each of these cases, contract terms were determined in private negotiations between buyers and sellers, and non-price considerations related to the quality of the products and the reputation of the suppliers were important.

The model that I develop here is designed to support economic predictions in such settings, using data that commonly become available to antitrust authorities during merger investigations. It fills a void in the academic literature. While much is known about simula-

[4]The third fully litigated merger challenge, H&R Block/TaxAct, involved the sale of tax preparation software to consumers.

tion techniques for consumer products industries, and those techniques often are applied in merger cases, much less research has tackled business-to-business transactions in a tractable manner. This deters the use of simulation in procurement markets. As Werden and Froeb (2006) argue, "a merger simulation should not be given significant weight by an enforcement agency or court unless the oligopoly model at the heart of the simulation fits the industry."

The model follows the established roadmap for merger simulation with consumer products. Namely, it specifies a mechanism that dictates competitive interactions, takes as given the quality of suppliers/products, and then uses parametric assumptions to translate data on market shares, prices, and marginal costs into price predictions.[5] One particular distributional assumption allows for calibration with market shares together with the price and marginal cost of a singe firm. Data on the prices and costs of other firms then can provide a first check on the parametric assumptions. Such data are available in most merger investigations, at least after the most preliminary stages. A second check on the model can be conducted by examining whether win/loss data and customer surveys, which are obtained in many investigations from the merging parties' documents, are consistent with the substitution patterns that arise given the maintained distributional assumptions.[6] With much more data, the model can be relaxed to accommodate arbitrary customer substitution, along the lines of Berry, Levinsohn and Pakes (1995) and Nevo (2001). This extension is unlikely to be feasible for merger enforcement except in exceptional cases, due to both data and time constraints, but it remains interesting from an academic standpoint.

3 The Model

3.1 Framework

Consider a market with many buyers and a finite number of suppliers. Let the utility that a buyer i receives from supplier $j \in N := \{1, \ldots, n\}$ be given by $u_{ij}(v, p) = v_{ij} - p_i$, where v_{it} is the value derived from the job and p_i is the payment to the supplier. Further let the marginal cost of supplier j when selling to buyer i be c_{ij} such that profit is given by $\pi_{ij}(c, p) = p_i - c_{ij}$. The difference between the value and the marginal cost is the surplus that can be created in a transaction between a specific buyer and seller. Denote surplus as $w_{ij}(v, c)$ such that $w_{ij}(v, c) = v_{ij} - c_{ij}$.

[5]The mechanism in many consumer products industries is Nash Bertrand competition. Here is the mechanism is the second-score auction, for which suppliers have a dominant strategy.

[6]Win/loss data catalog the results of a supplier's sales opportunities.

Buyers observe fully the values of each supplier but not suppliers' marginal costs. This information structure prevents buyers from extracting all available surplus by making a take-it-or-leave it offer. Instead, consider a second score auction scheme in which each supplier receives a score based on the value it can provide and an offer that reflects the lowest price at which it is willing to transact. The score that buyer i assigns to supplier j is linear and additively separable in the value and the offer, such that $S_{ij} = v_{ij} - b_{ij}$ where S_{ij} is the score and b_{ij} is the offer. Supplier j is selected if and only if $S_{ij} > \max_{k \in N} S_{ik}$.

The transaction price is determined such that the buyer's utility equals the surplus that could have been created by transacting with the second-best supplier. Letting supplier j be the first-best supplier, the resulting condition can be written

$$v_{ij} - p_i = \max_{k \neq j}\{v_{ik} - b_{ik}\} \tag{1}$$

or, equivalently,

$$p_i = v_{ij} - \max_{k \neq j}\{v_{ik} - b_{ik}\}. \tag{2}$$

Each supplier observes its own marginal cost and value when submitting its offer, but not the marginal costs or values of other suppliers. It is a dominant strategy for suppliers to submit offers at marginal cost, such that $b_{ij} = c_{ij}$ for all i, j, which is immediate from Vickrey (1961) after applying a change in variables. I assume hereafter that suppliers submit offers in accordance with this dominant strategy. This second-score auction is efficient in that the socially optimal outcome is realized. In alternative frameworks that allow suppliers to choose their value, efficiency still can be realized through quasi-linear scoring functions (e.g., Che 1993; Asker and Cantillon 2010).

The economic variables of interest can be expressed in terms of surplus. Supplier j is selected if and only if it provides the greatest total surplus, i.e. if and only if $w_{ij} = \max_k\{w_{ik}\}$. The price that arises is such that buyer utility equals the surplus that could have been created from the second-best supplier:

$$p_i = v_{ij} - \max_{k \neq j}\{w_{ik}\}, \tag{3}$$

assuming without loss of generality that supplier j is selected. This price determines the split of surplus between the buyer and the selected supplier. The profit of the first-best supplier, which equals price minus marginal cost, also can be expressed in terms the surplus it creates in excess of the surplus that could have been created by the second-best supplier:

$$\pi_{ij} = w_{ij} - \max_{k \neq j}\{w_{ik}\}, \tag{4}$$

letting supplier j be selected. Analogously, buyer utility equals the surplus that could have been created from the second-best supplier:

$$u_{ij} = \max_{k \neq j}\{w_{ik}\}. \tag{5}$$

In procurement settings, some buyers may elect not to purchase from any of the n suppliers. The framework can be adjusted to accommodate this by the incorporation of an outside option. Here I assume that the outside option, $j = 0$, provides value of v_{i0} and costs the buyer c_{i0}, for an available surplus of w_{i0}. The price that arises when the outside option is selected, following equation (3), can be interpreted either as a payment to a non-market participant or as a transfer price that governs intra-firm transactions.

3.2 Stochastic properties

Let surplus be exogenous and stochastic. Thus, different buyers may select different suppliers and the obtained prices, utility and profit are all random variables. Let the surplus that supplier j can provide in a transaction with buyer i be drawn independently from the distribution $G(\cdot|\boldsymbol{\theta}_j)$, where $\boldsymbol{\theta}_j$ is a vector of supplier-specific parameters related to marginal costs or values. Finally, let the maximum surplus available from any given set A of suppliers, denoted $z_A = \max_{k \in A}\{w_{ik}\}$, have the distribution $H_A(\cdot;\boldsymbol{\theta})$, and let the first moments of $G(\cdot|\boldsymbol{\theta}_j))$ and $H_A(\cdot;\boldsymbol{\theta})$ be finite.

Underlying the stochastic process in surplus is random variation in values, marginal costs, or both. Assume that values and marginal costs are drawn from (potentially degenerate) distributions with finite first moments. The results obtained do not require further knowledge of the value and cost distributions, given knowledge of the surplus distribution. Thus the model nests as special cases (i) markets with homogeneous products and supplier-specific marginal cost distributions, as in Waehrer and Perry (2003); (ii) markets with heterogeneous product values and common marginal costs; and (iii) markets with both heterogeneous product values and supplier-specific marginal cost distributions.

General expressions for the selection probabilities and the expected prices, buyer surplus, and profit that arise given the structure of the model can be obtained without placing further restrictions on $G(\cdot|\boldsymbol{\theta}_j)$ and $H_A(\cdot;\boldsymbol{\theta})$. The *ex ante* probability that supplier j is selected by buyer i equals the probability that its total surplus exceeds the maximum total surplus of other suppliers:

$$s_j(\boldsymbol{\theta}) = 1 - \Pr\left(w_{ij} < z_{\{k\neq j\}}; \boldsymbol{\theta}\right). \tag{6}$$

Conditional on supplier j being selected, expected variable profit equals

$$E\left[\pi_j |\, w_{ij} > z_{\{k\neq j\}}; \boldsymbol{\theta}\right] = E\left[z_N; \boldsymbol{\theta}\right] - E\left[z_{\{k\neq j\}}; \boldsymbol{\theta}\right] \tag{7}$$

and expected buyer utility equals

$$E\left[u_i |\, w_{ij} > z_{\{k\neq j\}}; \boldsymbol{\theta}\right] = E\left[z_{\{k\neq j\}}; \boldsymbol{\theta}\right]. \tag{8}$$

Finally, the expected price conditional on supplier j being selected equals

$$E\left[p_i |\, w_{ij} > z_{\{k\neq j\}}; \boldsymbol{\theta}\right] = E\left[v_{ij} | w_{ij} > z_{\{k\neq j\}}; \boldsymbol{\theta}\right] - E\left[z_{\{k\neq j\}}; \boldsymbol{\theta}\right]. \tag{9}$$

Alternatively, the expected conditional price can be formulated based on marginal costs. Adding and subtracting expected marginal costs, and substituting based on equation (7), yields an expression in terms of marginal costs and a markup term:

$$E\left[p_i |\, w_{ij} > z_{\{k\neq j\}}; \boldsymbol{\theta}\right] = E\left[c_{ij} | w_{ij} > z_{\{k\neq j\}}; \boldsymbol{\theta}\right] + E\left[\pi_j | w_{ij} > z_{\{k\neq j\}}; \boldsymbol{\theta}\right]. \tag{10}$$

Thus, whereas expected profit and buyer utility depend only on surplus, expected price depends both on surplus and on expected costs and values. Equations (7), (8), and (10) are fundamental to the analysis of mergers.

4 The Effect of Mergers

How mergers manifest depends on structure and magnitude of suppliers' fixed costs, which thus far have not been incorporated into the model. If fixed costs are sufficiently small then the merging firms likely find it profitable to retain both of their products and subsequently offer to maximize profit from their product portfolio. If instead suppliers incur separate fixed costs for each product (e.g., due to brand advertising) and these fixed costs are sufficiently large then the merging firms likely find it profitable to discontinue one of their products. I analyze these two cases in turn.

4.1 Retention of all products

I assume that the merging suppliers observe the costs and values of their products when submitting offers, but not the costs or values of non-merging suppliers. The information available to non-merging suppliers is unchanged. Under this information structure, it is a dominant strategy for the merging suppliers to offer their higher-surplus product at cost and to withhold their lower-surplus product.[7] Which product of the merging suppliers is higher- or lower-surplus remains stochastic and specific to the particular buyer in question. It follows immediately that, because the withheld product always creates inferior surplus, the merger affects neither the identity of the selected supplier nor the realized surplus. Further, aggregating across buyers, the *ex ante* probability that any supplier is selected remains as defined in equation (6).

The merger does affect price, however, whenever the surplus that can be created by the merging suppliers' products both exceed the surplus that can be created by the products of all non-merging suppliers. Consider a merger of suppliers j and n. The price change that arises when the products of suppliers j and n can create the most and second-most surplus, respectively, is given by

$$\Delta p_i = w_{ik} - \max_{k \neq j,n}\{w_{ik}\}, \tag{11}$$

reflecting that product n is withheld and no longer suppresses the price. The expected price change conditional on the product of supplier j creating the most total surplus is given by

$$E\left[\Delta p_i | w_{ij} > z_{\{k\neq j\}}; \boldsymbol{\theta}, \right] = E\left[z_{\{k\neq j\}} - z_{\{k\neq j,n\}}; \boldsymbol{\theta}\right]. \tag{12}$$

Finally, the unconditional expected change in price, keeping in mind that either j or n could be the higher-surplus product, takes the form

$$E\left[\Delta p_i; \boldsymbol{\theta}\right] \;\; = \;\; s_j(\boldsymbol{\theta}) E\left[z_{\{k\neq j\}} - z_{\{k\neq j,n\}}; \boldsymbol{\theta}, \right] + s_n(\boldsymbol{\theta}) E\left[z_{\{k\neq n\}} - z_{\{k\neq j,n\}}; \boldsymbol{\theta}\right]. \tag{13}$$

This latter expression incorporates that price does not change if the merging firms' higher-surplus product is not selected because, in that case, the withholding of the lower-surplus

[7] It is natural to question whether, in practice, the merging supplier would have sufficient confidence in its knowledge of the buyer's preferences to strategically withhold a product from the auction. Recall, though, that the second-score auction is equivalent to a bargaining scenario in which buyers play suppliers off against each other, up to the point that the utility offered by the highest-surplus supplier cannot profitably be matched by the next best supplier. Perfect information is not needed to generate the results in this section. Instead, what is required is that buyers cannot play off the merged entity against itself or, equivalently, that the merging suppliers successfully internalize the effect of competition between their products.

product does not eliminate a binding constraint on prices. The higher expected price that arises due to the merger represents a transfer of buyer utility to supplier profit. Because the identities of the selected suppliers are unchanged, the efficiency of the auction is unaffected.

4.2 Discontinuation of a product

This section analyzes mergers after which the product of one merging supplier is discontinued for all buyers. In this case, the post-merger environment is identical to the pre-merger environment except that there is one fewer supplier. Thus, all suppliers have the dominant strategy of bidding at marginal cost. The ramifications of a merger are twofold: First, for some buyers the merger eliminates the most preferred supplier and thereby reduces utility and surplus. While prices can increase or decrease, these buyers are strictly worse off. Second, for other buyers the merger eliminates the second-best supplier and thereby increases price and decreases utility, while leaving surplus unchanged.

Subcase 1. Consider first scenarios in which the buyer's preferred product is discontinued. Some additional notation is necessary: we denote as $s^*_{ijk}(\boldsymbol{\theta})$ the probability that buyer i's second-preferred choice is supplier j given that the most-preferred supplier is supplier k. Let the merging suppliers be j and n, and suppose the product of supplier j is discontinued. Since this preferred product initially offered the most surplus, discontinuation reduces the available surplus. The expected lost surplus equals

$$E[\Delta w_i | w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}] = E[z_{\{k \neq j\}}; \boldsymbol{\theta}] - E[z_N; \boldsymbol{\theta}]. \tag{14}$$

Buyer utility is determined by the surplus offered by the second-preferred product. Thus, discontinuation reduces buyer surplus and the expected change equals

$$E\left[\Delta u_i | s_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] = \sum_{l \neq j} s^*_{ilj}(\boldsymbol{\theta}) E\left[z_{\{k \neq j, l\}}; \boldsymbol{\theta}\right] - E\left[z_{\{k \neq j\}}; \boldsymbol{\theta}\right]. \tag{15}$$

The merging firms can gain or lose variable profit, depending on the realized distribution of surplus across suppliers. The expected change equals

$$\begin{aligned} E\left[\Delta \pi_i | S_{ij} > Z_{\{k \neq j\}}; \boldsymbol{\theta}\right] &= E[z_{\{k \neq j\}}; \boldsymbol{\theta}] - E[z_N; \boldsymbol{\theta}] \\ &+ s^*_{inj}(\boldsymbol{\theta})(E\left[z_{\{k \neq j\}} - z_{\{k \neq j, n\}}; \boldsymbol{\theta}\right]), \end{aligned} \tag{16}$$

reflecting the lost profit on supplier j's product as well the possibility of recoupment through the selection of supplier n's product. Increases in profit arise when (i) the merging suppliers' products offer high and similar magnitudes of surplus and (ii) non-merging suppliers' products offer less total surplus. Then discontinuation eliminates an important pricing constraint but does not substantially reduce surplus.

Subcase 2. Consider now scenarios in which the discontinued product is not preferred by the buyer. Again let the merging suppliers be j and n, and suppose the product of supplier j is discontinued. Surplus is unaffected because the highest-surplus product remains in the market. The merger does affect prices, and thus the split of surplus, whenever the discontinued product provides the second-highest surplus. The expected change in price, conditional on j providing the second-highest surplus, equals

$$E\left[\Delta p_i |\, z_N > s_{ij} > z^*_{\{k \neq j\}}; \boldsymbol{\theta}\right] = \sum_{l \neq j} \frac{s_l(\boldsymbol{\theta})}{1 - s_j(\boldsymbol{\theta})} E\left[z_{\{k \neq j, l\}} - z_{\{k \neq l\}} |\boldsymbol{\theta}\right], \tag{17}$$

where we use z^*_A to denote the second-highest surplus in the set A.

5 Distributional Assumptions

5.1 Location-scale and power-related distributions

The predictions above can be sharpened by imposing distributional assumptions on the $G(\cdot|\boldsymbol{\theta}_j)$, the distribution of surplus. I consider first the location-scale family of distributions, and then discuss briefly power-related distributions. With the location-scale distributions, let the vector of supplier-specific parameters be composed of the location parameter δ_j and a scale parameter σ that is common across suppliers, such that $\boldsymbol{\theta}_j = (\delta_j, \sigma)$. Distributions in this family include the Gumbel, normal, uniform and logistic distributions.

The restriction to the location-scale family allows surplus to be decomposed such that

$$w_{ij} \equiv v_{ij} - c_{ij} = \delta_j + \epsilon_{ij} \tag{18}$$

where the stochastic term ϵ_{ij} is drawn from some distribution $F(\cdot;\sigma)$. One location parameter is not identifiable and it is conventional to impose the normalization $\delta_0 = 0$. Because the surplus of each supplier is stochastic, the maximum surplus obtained from any set A of suppliers also is stochastic. Let this maximum have the distribution $H_A(\cdot;\boldsymbol{\delta}_A,\sigma)$, where $\boldsymbol{\delta}_A$ is a vector that contains δ_j for all $j \in A$. The stochastic draws are independent and identically distributed so that the distributions $H_A(\cdot;\boldsymbol{\delta}_A,\sigma)$ and $F(\cdot;\sigma)$ are related according to:

$$\max_{j\in A}\{w_{ij}\} = H_A(x;\boldsymbol{\theta}) = \prod_{j\in A} F(x-\delta_j;\sigma) \tag{19}$$

This equation is useful because the effects of mergers on expected price, utility, and profit all depend on the expected maximum surplus provided by specific sets of suppliers.

Closed-from solutions can be obtained for the specific case of the Gumbel distribution.[8] There the expected maximum surplus provided by the set A equals the inclusive value:

$$E\left[z_A;\boldsymbol{\delta}_A,\sigma\right] = \int_{-\infty}^{\infty} x dH_A(x;\boldsymbol{\delta}_A,\sigma) = \sigma \ln \sum_{j\in A} \exp\left(\frac{\delta_j}{\sigma}\right) \tag{20}$$

Further, the selection probabilities of each firm take the form

$$s_j(\boldsymbol{\delta},\sigma) = \frac{\exp(\delta_j/\sigma)}{\sum_{k=0}^{J}\exp(\delta_k/\sigma)} \tag{21}$$

Thus, provided values are distributed by the Gumbel distribution, analytical solutions are available for selection probabilities, expected profit, and expected prices.

This framework nests as a special case the model of Froeb, Tschantz and Crooke (1998, 2000). There it is assumed that (i) suppliers have heterogeneous marginal costs but are otherwise homogeneous, so that the value that buyers gain from a transaction does not vary across suppliers, and (ii) that suppliers' marginal costs have the Gumbel distribution. The first assumption can be accommodated in our framework by applying a change-of-variables to equation (18) and the location parameters; equations (20) and (21) then provide equations that can be used to predict the price effects of mergers.

Next consider the family of power-related distributions invoked in Froeb, Tschantz and Crooke (1999) and Waehrer and Perry (2003). Let the supplier-specific parameter represent the "capacity" of the supplier and define $\widehat{\delta} = \sum_j \delta_j$ as the total capacity of the suppliers. Capacity here can be interpreted as a summary measure of all economic factors that affect suppliers' ability to create surplus. Further, let the distribution $G(\cdot|\boldsymbol{\theta}_j)$ take the form

$$G(x|\delta_j) = 1 - [1-F(x)]^{\delta_j} \tag{22}$$

where $F(\cdot)$ belongs to the family of power-related distributions. Thus, capacity can be

[8] The Gumbel distribution often is referred to in the economics literature as the extreme value type I distribution.

interpreted as the number of draws that a supplier receives from the distribution F, where the highest draw determines the surplus that the supplier creates.

A number of useful properties arise, among which is that the probability distribution of the highest draw received by any set of suppliers A depends only on the total capacity of suppliers in the set. Defining the total capacity of a set A of suppliers as $\delta_A = \sum_{j \in A} \delta_j$, it is the case that

$$G(x|\delta_A) = 1 - [1 - F(x)]^{\delta_A} \tag{23}$$

This makes it tractable to obtain numerically the expected maximum surplus created by any set of suppliers – a key input to evaluated the effects of mergers within framework introduced here. Another useful property is that the selection probability for any supplier j takes has the closed-form expression $s_j(\boldsymbol{\delta}) = \delta_j / \widehat{\delta}$ as the fraction of total capacity that is held by the firm. These properties are well analyzed in Waehrer and Perry (2003), and I refer readers to that article for more details.

5.2 More on the Gumbel distribution

Among the distributions enumerated above, the Gumbel distribution is the most tractable for empirical work, and I extend some of the mathematics here. The Berry (1994) inversion applies so that the location parameters can be expressed as a linear function of differences in log selection probabilities:

$$\log(s_j) - \log(s_0) = \delta_j / \sigma \tag{24}$$

Further, equations (7), (10), (20), and (21) can be combined to yield an expression for expected price that depends on expected marginal cost and a simple markup term that depends only on the selection probability:

$$E\left[p_i | \, w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] = E\left[c_{ij} | w_{ij} > z_{\{k \neq j\}}; \boldsymbol{\theta}\right] + \sigma \log\left(\frac{1}{1 - s_j}\right) \tag{25}$$

The above equations allow the model to be fully calibrated with (i) market shares and (ii) the average price and average cost of a single firm.

The simulation results for Bazaarvoice/Power Review presented in the introduction rest on such calculations. I use market shares of 40% for Bazaarvoice, 28% for Power Reviews, 3% for other vendors and 28% for the in-house build, based on expert witness testimony summarized in the opinion of the district court. The average sales price and cost of goods sold ("COGS"), from the 2013 Annual Report of Bazaarvoice, are \$154,000 and \$101,000, respectively, and I take these as measures of average price and cost.[9] This is sufficient for calibration. If all products are retained post-merger then the Bazaarvoice price increases \$65 (42%) and the Power Reviews price increases \$84. If instead the Power Reviews product is discontinued, the total utility loss for Power Reviews clients is \$79,000 annually.

That so little is required for merger simulation is due to the restrictions that arise with the Gumbel distribution. Most famously, models such as this one exhibit independence of irrelevant alternatives (IIA), meaning that for any suppliers j and k and any subset B of A,

$$\frac{s_j(B)}{s_k(B)} = \frac{s_j(A)}{s_k(A)}$$

The IIA property implies that consumers substitute between suppliers in proportion to market share. For example, suppose that two suppliers j and k have selection probabilities of 0.1 and 0.2, respectively. If another supplier experiences a decrease in its expected surplus, due to reductions in value or increases in cost, then the selection probability of supplier k increases by twice that of supplier j.

The restrictions of the model can be evaluated with somewhat more information. Because equation (25) characterizes markups for every supplier, data on average prices and average costs for other firms can provide a first check on the model. Such information is available in most antitrust investigations, at least after the most preliminary stages. The cost data typically are of higher quality than what is available for academic work because firms have a strong incentive to understand their cost structure, and this tends to be reflected in the normal-course business documents that become available to antitrust authorities. A second check on the model can be conducted by examining whether win/loss data or customer substitution surveys, which are obtained in many investigations from the merging parties' documents, are consistent with the IIA property implied by the Gumbel distribution.

With even more data, the restrictions of the model can be relaxed to accommodate arbitrary patterns of customer substitution, along the lines developed in Berry, Levinsohn and Pakes (1995) and Nevo (2001) for consumer products industries. The starting point is the surplus equation because surplus dictates buyer choice (rather than indirect utility as with consumer products). Suppose, for the purposes of illustration, that the marginal cost

[9] Many caveats apply to the use of accounting data in this manner. Both the average sales price and the COGS incorporate monies associated with products other than product ratings and reviews. Further, they are calculated as straight averages, across all clients, so a mismatch exists with the market shares, which are based on large clients. The COGS has the additional problem that it may incorporate expenditures that are not marginal. Additionally, no attempt has been made to calculate the net present value of a client relationship which, in this case, probably aligns better with the appropriate notion of markup.

of each firm is fixed across buyers, while value can be decomposed into a function of product characteristics and buyer-specific taste parameters. Then surplus is given by

$$w_{ij} = \boldsymbol{x}_j' \boldsymbol{\beta}_i - mc_j + \xi_j + \epsilon_{ij}, \tag{26}$$

where $\boldsymbol{x}_j'$ is a vector of observable product characteristics, $\boldsymbol{\beta}_i$ is a vector of buyer-specific taste parameters, ξ_j is the mean buyer valuation of unobservable product characteristics, and ϵ_{ij} is a consumer-specific taste shock. Further, let the taste parameters be a function of observed and unobserved buyer characteristics:

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\Pi} \boldsymbol{D}_i + \boldsymbol{\Sigma} \boldsymbol{\nu}_i, \tag{27}$$

where $\boldsymbol{D}_i$ and $\boldsymbol{\nu}_i$ are vectors of observed and unobserved buyer characteristics, respectively.

This formulation is nearly equivalent to the models of Berry, Levinsohn and Pakes (1995) and Nevo (2001), and I refer readers to those articles for details on estimation. The difference is that marginal costs, rather than prices, govern buyer choices. While this changes little about the mechanics of estimation, it does limit applications to settings in which costs can be measured with some confidence. Further, because estimation requires considerable cross-sectional or time-series variation in market shares, and also is computationally expensive, this extension is infeasible for the antitrust context outside exceptional cases. Its value is more likely as a framework for empirical research on business-to-business markets.

6 Extensions

6.1 Nash bargaining

The second-score auction is equivalent to a specific form of bargaining. Buyers play suppliers off against each other, up to the point at which the utility offered by the highest-surplus supplier cannot be matched profitably by the next best supplier. At that point, buyers have no more leverage and negotiations end. This conveys tractability to the analysis and is reasonable for many markets with either (i) merging suppliers that are much larger than most buyers, as in the AT&T/T-Mobile business products, or (ii) buyers that have sizable negotiating costs, as in Bazaarvoice/Power Reviews.[10] The model nonetheless can be generalized

[10] In Bazaarvoice/Power Reviews, the DOJ's expert witness testified that the prices of ratings and reviews platforms are sufficiently low that most customers (e.g., Staples or Home Depot) have little incentive to monitor or study the ratings and reviews market. This testimony is summarized in the opinion of the

to incorporate more extensive bargaining between buyers and the highest-surplus supplier.

Consider the "Nash-in-Nash bargaining" solution proposed in Horn and Wolinsky (1988), which is the theoretical underpinning of several recent empirical articles on bargaining (e.g., Crawford and Yurukoglu 2012; Grennan 2013; Gowrisankaran, Nevo, and Town 2014). In the context of this model, the solution requires that the transfer prices negotiated between a buyer and each supplier represent the solutions to bilateral Nash bargaining, conditional on all the other negotiated prices. This changes little for inferior suppliers – in subgame perfect equilibrium, they offer prices that transfer all available surplus to the buyers, yet are not selected. The highest-surplus supplier, however, no longer retains as profit all the surplus it creates in excess of what could have been created by the second-best supplier. Rather, a bargaining parameter determines the split of this excess surplus.

The Nash-in-Nash solution, letting j be the highest-surplus supplier for buyer i, is given by

$$p_i = \arg\max_p (p - c_{ij})^{\alpha}(v_{ij} - p - z_{\{k \neq j\}})^{1-\alpha} \tag{28}$$

subject to the restriction $p \in [c_{ij},\ v_{ij} - z_{\{k \neq j\}}]$, where $\alpha \in [0, 1]$ is the bargaining parameter. The first term on the right hand side is the profit that the supplier receives from the transaction, less the profit that the supplier would receive absent the transaction, which is zero. The second term is the buyer utility from the transaction less the buyer utility that could be obtained, absent the transaction, from the next best supplier.

The bargaining parameter determines the split of excess surplus. If $\alpha = 1$, then the supplier has all the bargaining power, and $p_i = v_{ij} - z_{\{k \neq j\}}$. This is the highest price at which the buyer prefers supplier j over the next best supplier. It is also the solution of the second-score auction. If instead $\alpha = 0$, then the buyer has all the bargaining power, and $p_i = c_{ij}$. This is the lowest possible price at which supplier j is willing to transact. Mergers are uninteresting in this case because suppliers cannot exercise market power. For $\alpha \in (0, 1)$, the highest-surplus supplier and the buyer split the surplus that is in excess of what could have been created by the second-best supplier.

Extending the derivations in Section 4 to account for bargaining is straight-forward because bargaining does not affect choice probabilities. With both the second-score auction and the Nash-in-Nash bargaining model, the highest-surplus supplier always wins. In both models, price effects arise if and only if the products of the merging suppliers are create the most and second-most surplus, assuming that all products are retained post-merger.

district court.

6.2 First score auctions and Bertrand pricing

I consider first-score auctions in this final extension. This formulation of negotiation is less interesting for two reasons: First, it contains none of the "back-and-forth" between buyers and suppliers that is common in procurement settings. The first-score auction requires that buyers commit, in advance of the auction, not to negotiate after offers are received. This is difficult in most business-to-business markets. Second, the economics of first-score auctions resemble closely the economics of consumer products markets and Bertrand pricing models, which are well developed in the literature. These connections have been highlighted in previous research (e.g., Einav 2003).

As before, let each supplier j submit an offer b_{ij} to buyer i with knowledge of its own value and marginal cost but without knowledge of its competitors' values and costs. The supplier with the highest score, $S_{ij} = v_{ij} - b_{ij}$ again wins the contract. The price of the first-score auction, however, is the offer of the highest-score supplier. In a slight abuse of notation, let $s_{ij}(\boldsymbol{b}_i)$ be the *ex ante* selection probability of supplier j given the vector of offers $\boldsymbol{b}_i = [b_{i1}, b_{i2}, \ldots]$. Then the profit maximizing offer of each supplier j satisfies the first order condition

$$b_{ij} = c_{ij} - \left(\frac{\partial s_{ij}(\boldsymbol{b}_i)}{\partial b_{ij}}\right)^{-1} s_{ij}(\boldsymbol{b}_i) \tag{29}$$

Offers equal marginal cost plus a markup term and, in equilibrium, the highest-surplus supplier wins the contract. Equation (30) is identical to the pricing equations that arise with Nash-Bertrand competition among single-product firms, and the extension to multi-product firms is trivial. A merger among suppliers changes the bidding calculus because the merged entity internalizes the effect of competition between its products. Considering a merger of suppliers j and n, the post-merger first order conditions for product j are given by

$$b_{ij}^{post} = c_{ij} - \left(\frac{\partial s_{ij}(\boldsymbol{b}_i^{post})}{\partial b_{ij}^{post}}\right)^{-1} s_{ij}(\boldsymbol{b}_i^{post}) - \left(\frac{\partial s_{ij}(\boldsymbol{b}_i^{post})}{\partial b_{ij}^{post}}\right)^{-1} \left(\frac{\partial s_{in}(\boldsymbol{b}_i^{post})}{\partial b_{ij}^{post}}\right) (b_{in}^{post} - c_{in}) \tag{30}$$

The effect of these new first order conditions on offers, again following the consumer products roadmap, can be simulated based on distributional assumptions along the lines discussed in Section 5. Alternatively, because the new term in the first order conditions enters quasi-linearly with a coefficient of unity, as does the marginal cost, in some cases the first order effect of the merger on offers can be calculated directly if pass-through data are available (Jaffe and Weyl 2013; Miller, Remer, Ryan and Sheu 2014).

7 Conclusion

This article has described a stochastic economic model of procurement markets that can be used to predict the effects of mergers between suppliers on market outcomes such as prices, buyer utility and supplier profit. Unlike the existing antitrust literature on auctions, it accommodates product differentiation among suppliers in addition to cost differentiation. Further, the model can be calibrated with data often available to antitrust authorities and thus may be of practical use in evaluating mergers between suppliers in procurement markets. That predictions can be made with limited information is due, in part, to the assumption that the value created by suppliers is exogenous and not subject to manipulation. Left for future research are the interesting questions of whether and how mergers affect market outcomes when supplier value is endogenous and, especially, non-contractible.

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